Zadatak: Neka su a,b i c stranice trougla i α, β i γ su naspramni uglovi, a R poluprečnik
opisane kružnice trougla. Dokazati:

a)Tangensnu teoremu: $\frac{a+b}{a-b}=\frac{tg\frac{\alpha+\beta}{2}}{tg\frac{\alpha-\beta}{2}}$

b) Molvajdove (Mallweid) teoreme: $\frac{a+b}{c}=\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}}$ i $\frac{a-b}{c}=\frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\gamma}{2}}$

Rješenje:

a) Dokaz tangensne teoreme slijedi iz sinusne teoreme:

$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}=2\text{R}$ ....(1)
iz (1) $\Longrightarrow a=2\text{R}\sin\alpha$ i $b=2\text{R}\sin\beta$ ....(2)

Koristeći (2) imamo: $\frac{a+b}{a-b}=\frac{2R\sin\alpha+2R\sin\beta}{2R\sin\alpha-2R\sin\beta}=\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}=\frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}}=\frac{tg\frac{\alpha+\beta}{2}}{tg\frac{\alpha-\beta}{2}}$

Dakle vrijedi: $\frac{a+b}{a-b}=\frac{tg\frac{\alpha+\beta}{2}}{tg\frac{\alpha-\beta}{2}}$

b) $\frac{a+b}{c}=\frac{2R\sin\alpha+2R\sin\beta}{2R\sin\gamma}=\frac{\sin\alpha+\sin\beta}{\sin\gamma}=\frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}=\frac{\sin\left(90^\circ-\frac{\gamma}{2}\right)\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}=\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}}$

Dakle vrijedi: $\frac{a+b}{c}=\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}}$

Dokaz druge tvrdnje pod b) uradite sami.